

# Cosmological Physics with Black Holes (and Possibly White Dwarfs)

Kristen Menou, Zoltan Haiman

*Department of Astronomy, Columbia University, 550 West 120th Street, New York, NY 10027, USA*

Bence Kocsis

*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA*

---

## Abstract

The notion that microparsec-scale black holes can be used to probe gigaparsec-scale physics may seem counterintuitive, at first. Yet, the gravitational observatory *LISA* will detect cosmologically-distant coalescing pairs of massive black holes, accurately measure their luminosity distance and help identify an electromagnetic counterpart or a host galaxy. A wide variety of new black hole studies and a gravitational version of Hubble’s diagram become possible if host galaxies are successfully identified. Furthermore, if dark energy is a manifestation of large-scale modified gravity, deviations from general relativistic expectations could become apparent in a gravitational signal propagated over cosmological scales, especially when compared to the electromagnetic signal from a same source. Finally, since inspirals of white dwarfs into massive black holes at cosmological distances may permit pre-merger localizations, we suggest that careful monitoring of these events and any associated electromagnetic counterpart could lead to high-precision cosmological measurements with *LISA*.

*Key words:*

---

## 1. Introduction

Essentially all astronomical measurements are performed via electromagnetic waves. The availability of accurate gravitational wave measurements within the next decade or so will thus be a significant development for astronomy. In particular, since the propagation of photons and gravitons could differ at a fundamental level, gravitational waves emitted by cosmologically-distant “space-time sirens,” such as coalescing pairs of massive

black holes, could be used as valuable new probes of physics on cosmological scales.

Black holes with masses  $\gtrsim 10^6 M_\odot$  are present at the center of numerous nearby galaxies (e.g. Kormendy & Richstone, 1995; Magorrian, 1998). As such galaxies collide over cosmic times, their central black holes coalesce, releasing  $\gtrsim 10^{58}$  ergs of binding energy in the form of gravitational waves (hereafter GWs). To measure the GWs emitted by these cosmologically-distant space-time sirens, ESA and NASA will build the Laser Interferometer Space Antenna, *LISA*<sup>2</sup>.

GWs emitted by black hole binaries have the unfamiliar property of providing a direct measure of the

---

<sup>1</sup> K. Menou gratefully acknowledges more than a decade of exchanges with Jean-Pierre Lasota on the physics of black holes (and definitely white dwarfs).

<sup>2</sup> <http://lisa.nasa.gov/>

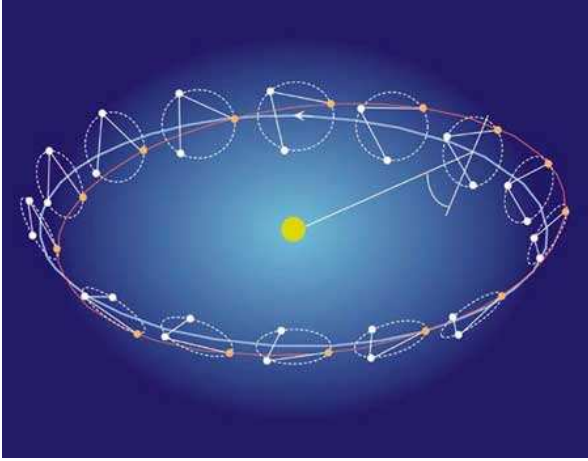


Fig. 1. Orbit of the LISA 3-base space interferometer, trailing the Earth around the Sun. A typical coalescing black hole pair observed for a year is well-modulated by the interferometer and thus accurately localized on the sky. [Credit: <http://lisa.nasa.gov/>]

luminosity distance,  $D_L$ , to the black holes, without extrinsic calibration. Owing to the highly coherent nature of GW emission (Schutz, 1986), the amplitude (or strain),  $h_{+ \times}$ , frequency,  $f$ , and frequency derivative,  $\dot{f}$ , of the leading order (quadrupolar) GW inspiral signal scale as

$$h_{+ \times}(t) \propto \frac{[(1+z)M_c]^{5/3} f^{2/3}}{D_L}, \quad (1)$$

$$\dot{f}(t) \propto [(1+z)M_c]^{5/3} f^{11/3}, \quad (2)$$

where  $+ \times$  represents the two transverse GW polarizations,  $M_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$  is the black hole pair “chirp” mass and  $z$  its redshift. Provided the GW source can be reasonably well localized on the sky, an extended observation of the chirping signal leads to precise measurements of  $h_{+ \times}$ ,  $f$ ,  $\dot{f}$  and thus  $D_L$ , independently. As illustrated in Fig. 1, LISA’s orbital configuration allows for a “triangulation” of GW sources on the sky, to within a solid angle  $\delta\Omega \sim 1 \text{ deg}^2$  typically (Cutler, 1998; Vecchio, 2004). This permits very accurate measurements, e.g. distances with errors  $\delta D_L / D_L < 1\%$  at  $z \lesssim 2$  typically (Cutler, 1998; Hughes, 2002; Vecchio, 2004; Lang & Hughes, 2006). Masses are independently determined to very high accuracy (typically  $\ll 1\%$ ; e.g., Hughes 2002)

## 2. Post- and Pre-Merger Localizations

In principle, the same sky localization that helps determine the distance to a source accurately can be used to find the host galaxy of a pair of merging black holes seen by LISA. The secure identification of the host galaxy would enable a wide variety of new galactic black hole studies (see §3.1).

Initially, the prospects for finding the host galaxy of a pair of merging black holes were considered to be poor, simply because of the large number of galactic candidates located in the  $\delta\Omega \sim 1 \text{ deg}^2$  LISA sky error-box (e.g., Cutler, 1998; Vecchio, 2004). Recently, however, this possibility has been reconsidered, with more optimistic conclusions (Holz & Hughes, 2005; Kocsis et al., 2006, 2007).

Given a cosmology, it is possible to translate the accurate luminosity distance measurement to the GW source into a narrow redshift slice in which the host galaxy must be located (Holz & Hughes, 2005; Kocsis et al., 2006). Various contributions to the redshift errors that arise in performing this conversion are shown in Fig. 2, for a representative equal-mass binary, as a function of the GW source redshift (Kocsis et al., 2006). At redshifts  $z \gtrsim 0.25$ , where most black hole binary sources are expected to be found, weak lensing errors due to line-of-sight inhomogeneities (on top of the smooth average cosmology) are the main limitation to an accurate determination of the redshift slice in which the host galaxy ought to be located.

Kocsis et al. (2006) have studied in detail the possibility that the three-dimensional information available (sky localization + redshift slice) could be used to single out a quasar, or any other unusually rare object (such as a star-bust galaxy), in the LISA error box, after coalescence. Finding such a statistically rare object post-merger would make it a good host galaxy candidate for the newly-coalesced pair of black holes.

However, it maybe much more advantageous to use a pre-merger strategy to identify the host galaxy of a pair of coalescing black holes seen by LISA. Indeed, one can use near real-time GW information on the sky localization, in combination with the accurate timing of the inspiral event, to predetermine well in advance where on the sky the merger is located. A unique host galaxy identification could then proceed through coordinated observations with traditional telescopes, by monitoring in real time the sky area for unusual electromagnetic emission, as

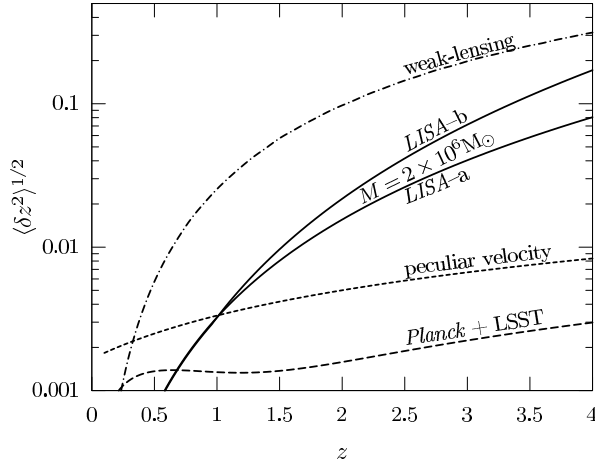


Fig. 2. Contributions to the error on the inferred redshift of an electromagnetic counterpart to a *LISA* coalescence event, as a function of redshift  $z$ , for  $m_1 = m_2 = 10^6 M_\odot$ . The intrinsic *LISA* error on the luminosity distance,  $d_L$ , is shown for two representative cases (a & b, solid lines). Errors due to the peculiar velocity of the source (for  $v = 500 \text{ km s}^{-1}$ ; short-dashed line), uncertainties on the background cosmology (long-dashed line), and errors due to weak lensing magnification (dash-dotted line) are also shown (see Kocsis et al. 2006 for details).

the coalescence proceeds.

A variety of mechanisms exist through which disturbed gas in the vicinity of black hole pairs will power electromagnetic emission during and after coalescence (Armitage & Natarajan, 2002; Milosavljevic & Phinney, 2005; Dotti et al., 2006; Bode & Phinney, 2007; MacFadyen & Milosavljevic, 2008). For example, at the time of coalescence,  $\gtrsim 10^{53}$  ergs of kinetic energy are delivered to the recoiling black hole remnant and its environment, for typical recoil velocities  $\gtrsim 100 \text{ km/s}$  (e.g., Baker et al., 2006, 2007; Campanelli et al., 2007; Herrmann et al., 2007; Schnittman & Buonanno, 2007). This may lead to detectable signatures (Lippai et al., 2008; Milosavljevic & Phinney, 2005) and permit the coincident identification of a unique host galaxy. The detailed nature of such electromagnetic counterparts remains largely unknown, however.

To a large extent, *LISA*'s ability to localize a long-lived source on the sky is related to the GW signal being modulated as a result of the detector's revolution and change of orientation when the constellation orbits around the Sun (Fig. 1). Even though most of the GW SNR accumulates during the final stages of inspiral/coalescence for typical GW sources, reasonably good information

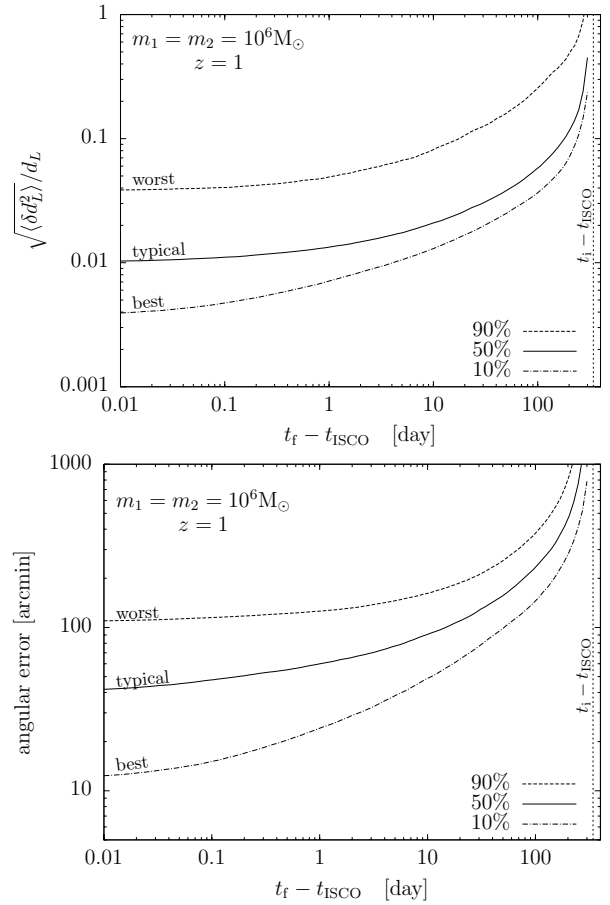


Fig. 3. Evolution with pre-merger look-back time,  $t_f - t_{\text{ISCO}}$ , of *LISA* source localization errors, for  $M = 2 \times 10^6 M_\odot$  and  $z = 1$ . The top panel shows luminosity distance errors and the bottom panel shows sky position angular errors (equivalent diameter,  $2\sqrt{ab}$ , of the error ellipsoid). Best, typical, and worst cases for random orientation events represent the 10%, 50%, and 90% levels of cumulative error distributions, respectively. Errors for worst case events effectively stop improving at a finite time before merger, even though the signal-to-noise ratio accumulates quickly at late times. Errors for best case events (especially the minor axis) follow the signal-to-noise ratio until the final few hours before merger (see Kocsis et al. 2007 for details)

on sky localizations must be available well before final coalescence since this information accumulates slowly, over the long signal modulation (orbital) timescale. Because of significant cross-correlations between sky localization and distance errors, it turns out that this argument is also largely valid for luminosity distance errors (Holz & Hughes, 2005; Kocsis et al., 2006, 2007).

Figure 3 shows the pre-merger time evolution of luminosity distance and angular sky localization errors for a representative black hole pair at  $z = 1$ .

Errors improve quickly at early times but their evolution slows down considerably at late times. According to both panels, even accounting for random orientations of various source and detector angles (shown as best, typical and worst cases), significant information is available days to weeks prior to the final coalescence (Kocsis et al., 2007). Including black hole spins in the analysis has been shown to result in significant improvements on the errors during the last few days to hours prior to coalescence (Lang & Hughes, 2008).

With the expected availability, by the time LISA is operational, of sensitive large field-of-view (FOV) astronomical instruments for weak lensing and supernova studies, it becomes interesting to estimate the amount of time prior to merger during which the LISA sky localization falls within the FOV of such an instrument. When this happens, continuous monitoring of the designated sky area, until final coalescence, becomes possible. Kocsis et al. (2007) have performed a detailed analysis of this possibility, using LSST and its  $10 \text{ deg}^2$  FOV as a reference. Figure 4 shows results for representative equal-mass binaries, as a function of their total mass and redshift. The various contours show that prospects for electromagnetic monitoring days to weeks before the coalescence are good for sources at redshifts  $z \lesssim 2$ –3. Monitoring for the best GW sources out to  $z \sim 5$ –7 may even be possible (Kocsis et al., 2007).

### 3. New Science with Electromagnetic Counterparts

#### 3.1. Galactic Black Hole Astrophysics

A large variety of new galactic black hole astrophysics would be enabled by successful identifications of the host galaxies of coalescing black hole pairs. We mention only a few possibilities here and refer the interested reader to Kocsis et al. (2006) and Kocsis et al. (2008) for additional discussions.

From the black hole masses, spins and binary orientation, all accurately constrained by the GW signal, one would be able to study the physics of the post-merger accretion flow onto the remnant black hole (Milosavljevic & Phinney, 2005; Dotti et al., 2006) with unprecedented accuracy. This would include precise constraints on the Eddington ratio of the accreting source, its emission and absorption geometries and possibly its jet phenomenology. Similarly, studies of the galactic host might tell

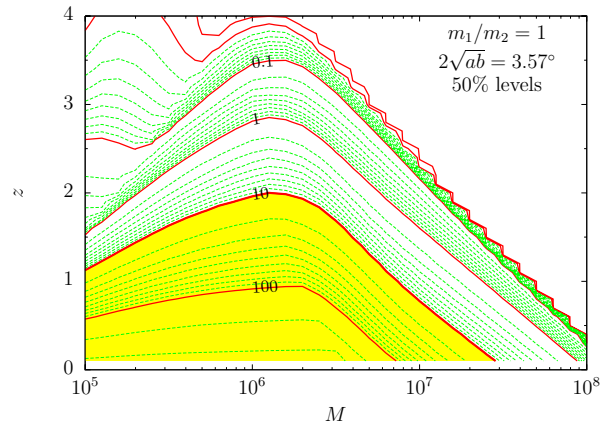


Fig. 4. Contours of advance warning times in the total mass ( $M$ ) and redshift ( $z$ ) plane for equal-mass black hole binaries ( $m_1/m_2 = 1$ ). The contours trace the look-back times at which the equivalent radius ( $2\sqrt{ab}$ ) of the LISA localization error ellipsoid first reaches an LSST-equivalent field-of-view ( $3.57^\circ$ ). These contours correspond to the 50% level of cumulative distributions for random orientation events. The contours are logarithmically spaced in days and 10 days is highlighted with a thick line (see Kocsis et al. 2008 for details).

us about the nature (dry/wet) and the timing of the galactic merger that resulted in the black hole binary coalescence. Finally, measuring velocity dispersions,  $\sigma$ , for several host galaxies, together with the black hole masses known from the GW signal, would allow us to accurately map the evolution of the  $M_{\text{bh}}-\sigma$  relation with cosmic time, at least for such transitional objects as the hosts of coalescing black hole pairs.

#### 3.2. Gravitational Hubble Diagram

Another consequence of successfully identifying the host galaxies of coalescing black hole pairs is the possibility to draw a gravitational Hubble diagram, i.e. one that relates the gravitational luminosity distances,  $D_L$ , of these GW sources to the electromagnetic redshifts,  $z$ , of their host galaxies.

One of the main interests of a gravitational Hubble diagram arises from its immunity to common systematics affecting electromagnetic measurements. Indeed, a gravitational Hubble diagram, which is based on gravitational distance measurements with self-calibrated sources, is not susceptible to any significant bias from absorption, scattering or reddening of GWs.

In practice, however, the value of such a diagram is limited by line-of-sight matter inhomogeneity.

geneities, which generate weak lensing uncertainties on the gravitational  $D_L$  measurement (see Fig. 2; Holz & Hughes, 2005; Kocsis et al., 2006; Dalal et al., 2006; Gunnarsson et al., 2006; Linder, 2008). While the lensing effect can in principle be averaged out over many random lines-of-sights, it may not be possible to do so for coalescing pairs of massive black holes if LISA merger event rates are modest (e.g., a few tens per year at  $z \lesssim 5$ ; Menou et al., 2001; Wyithe & Loeb, 2003; Sesana et al., 2004; Micic et al., 2007). Weak lensing errors on individual measurements amount to distance uncertainties ranging from  $\delta D_L/D_L \simeq 1\%$  at  $z = 0.5$  to  $\delta D_L/D_L \simeq 10\%$  at  $z = 5$  (e.g. Kocsis et al., 2006), which makes a gravitational Hubble diagram imprecise even at moderate redshifts. The extent to which LISA events can be used to draw a meaningful Hubble diagram will thus depend strongly on the actual distribution of massive black hole merger events with redshifts and the corresponding efficiency of host galaxy identifications. As we describe in §4, however, white dwarf spiraling into massive black holes may offer a practical avenue for precision cosmology with LISA.

### 3.3. Diagnostics of Modified Gravity

The possibility that the accelerated expansion of the Universe results from a failure of general relativity has fueled much theoretical work on large scale modifications of gravity over the past few years. Since building a satisfactory theory of modified relativistic gravity is a formidable task, any insight that can be gained from direct observational constraints on the linearized GW regime cannot be overlooked. LISA, with its ability to measure the GW signal from cosmologically-distant sources, may thus be one of our best probes of modified gravity on cosmological scales (Deffayet & Menou, 2007).

One may expect gravity modifications to contain a new length scale,  $R_c$ , beyond which gravity deviates from general relativity. In order to explain the observed accelerated expansion of the Universe, this scale is expected to be of the order of the current Hubble radius,  $H_0^{-1}$ . An existence proof of modifications of this type is given by DGP gravity (Dvali et al., 2000; Lue, 2006), a braneworld model with an infinite extra dimension.

Deffayet & Menou (2007) discuss the possibility that extra-dimensional leakage of gravity in DGP-like scenarios may lead to cosmologically-

distant GW sources appearing dimmer than they truly are, from the loss of GW energy flux to the extra-dimensional bulk. Indeed, in the presence of large distance leakage, flux conservation over a source-centered hypersphere requires that the GW amplitude scales with distance  $D$  from the source as

$$h_{+\times} \propto D^{-(dim-2)/2}, \quad (3)$$

where  $dim$  is the total number of space-time dimensions accessible to gravity modes. Thus, for  $dim \geq 5$ , it deviates from the usual  $h_{+\times}(D) \propto 1/D$  scaling. In principle, black hole merger events and associated host galaxies could thus reveal the leakage of gravity over scales of order a few Hubble distances, by comparison to purely electromagnetic Hubble diagrams, which are immune to such leakage effects.

This is only one of several possible modified gravity signatures in the GWs from cosmologically-distant sources (Deffayet & Menou, 2007). Another class of signatures is related to the GW polarization signal, with possibly additional polarizations beyond the two transverse quadrupolar ( $+\times$ ) modes of general relativity (e.g., Will, 2006). Signatures also exist in relation to the GW propagation velocity which, in modified gravity scenarios, can differ from the speed of light. In this respect, the possibility to time a cosmological GW, relative to an electromagnetic signal causally associated with the black hole merger, may offer unique diagnostics of large-scale modified gravity. This could reveal, for instance, that the phase of the GW signal deviates from general relativistic expectations, once propagated over cosmological distances.

Kocsis et al. (2008) have explored further the possibilities of measuring photon and graviton arrival times from a same cosmological source. A general difficulty with this approach is that there will be a systematic and a priori unknown delay in the emission of photons, relative to the emission of gravitons, since the former must causally lag behind the perturbing gravitational event. This difficulty could be overcome if it were possible to calibrate the relative timing of the photon and graviton signals at the source.

Prior to coalescence, gas present in the near environment of the black hole binary would be gravitationally perturbed in such a way that it could radiate a variable electromagnetic signal with a period closely matching that of the leading-order quadrupolar perturbation induced by the coalescing binary (see Fig. 5). This would help identify the electromagnetic counterparts of specific GW

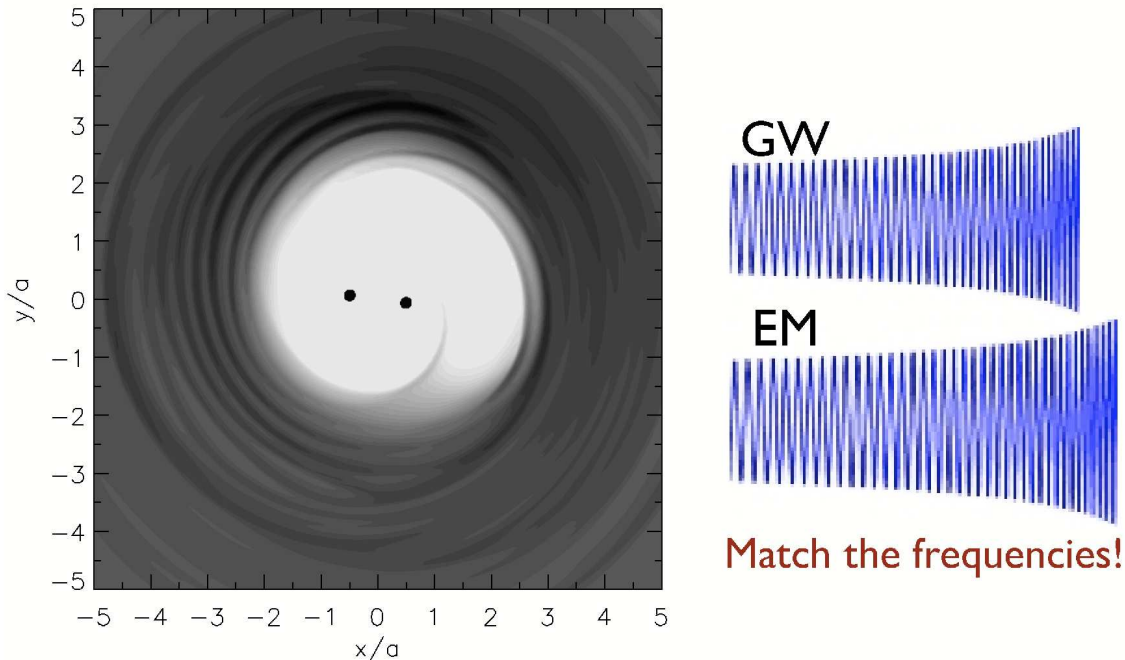


Fig. 5. [Left] From the numerical simulations of MacFadyen & Milosavljevic (2008): snapshot of a gaseous disk gravitationally-perturbed by the time-dependent quadrupolar potential of a central binary. [Right] For such a system, in the inspiral phase, one may expect an electromagnetic (EM) source varying in brightness at a frequency approaching that of the GW signal related to the quadrupolar perturbation. By matching the frequencies of the EM and GW signals, one could remove the delay in EM emission at the source, measure reliable offsets in arrival times between the two signals and possibly reveal drifts in frequency of the GW signal when it is propagated over cosmological distances.

events. In addition, it may be possible to match the variability frequencies of the electromagnetic and GW signals. The offset in phase between the Fourier components of the two signals with similar frequencies could be used to effectively calibrate the intrinsic delay in electromagnetic emission at the source. Late inspiral and coalescence can be tracked via the GW signal, so that the relative timing of the gravitational and electromagnetic signals may be known to within a fraction of the binary’s orbital time. Any drift in arrival-time with frequency between the gravitational and electromagnetic chirping signals, as the source spans about a decade in GW frequency during the last 2 weeks before merger, could then be attributed to a fundamental difference in the way photons and gravitons propagate over cosmological distances. For instance, such a drift could occur if the graviton is massive, resulting in a frequency-dependent propagation velocity (e.g. Berti et al., 2005; Will, 2006; Kocsis et al., 2008). This tracking possibility is illustrated graphically in Fig. 5.

Interestingly, while Lorentz invariance has been extensively tested for standard model fields, Lorentz

symmetry could be violated in the gravity sector, especially on cosmological scales (e.g., Csáki et al., 2001; Chung, Kolb & Riotto, 2002). With a good enough understanding of the source, electromagnetic counterparts to black hole binary mergers may offer unique tests of Lorentz violations in the gravity sector, via the opportunity to match and track the gravitational and electromagnetic signals in frequency and phase. It may be possible, as the black hole binary decays toward final coalescence, spanning a range of frequencies, to measure the delays in graviton vs. photon arrival times as a function of increasing frequency of the chirping signal. The consistency expected if Lorentz symmetry is satisfied in the gravity sector could be tested explicitly for gravitons propagated over cosmological scales. To have any chance to perform such new tests of gravitational physics, one will need to identify the electromagnetic counterparts of coalescing pairs of massive black hole binaries as early as possible. This may be one of the strongest motivations behind ambitious efforts to localize these rare, transient events well before final coalescence.

#### 4. New Possibilities with White Dwarf Inspirals

As seen before, a large number of sources must be accumulated to turn a gravitational Hubble diagram into a high precision tool for cosmology. White dwarf frequently spiraling into (moderately) massive black holes may offer unique opportunities in this respect.

An additional goal of the LISA mission is the detection of GWs emitted by compact objects being captured by massive black holes (the so-called Extreme Mass Ratio Inspirals, or EMRIs). Barack & Cutler (2004) present a detailed parameter estimation analysis for this class of LISA events and show that they could be detected with good SNR out to a distance  $\sim 1$  Gpc. In addition, these authors found that the sky localization errors for these events ( $\sim 10^{-3}$  steradians at  $\sim 1$  Gpc) are comparable to the case of black hole binary merger events. No electromagnetic counterpart is expected from EMRIs of dense neutron stars or stellar-mass black holes. On the other hand, partial (or total) disruptions of inspiraling low-density white dwarfs (WDs) could produce such counterparts and thus help identify host galaxies for such events. To the best of our knowledge, the possibility to find the electromagnetic counterparts of this subclass of LISA EMRIs and use them for precision cosmology has not been previously discussed in the literature.

As a result of its relatively low density, a typical WD suffers complete disruption around a non-spinning black hole of mass  $\lesssim 10^5 M_\odot$ . This mass limit is increased to perhaps  $\lesssim 10^6 M_\odot$  if the black hole is spinning rapidly (Ivanov & Chernyakova, 2006; Rathore et al., 2005). Even if the WD were not disrupted, some level of partial shedding of its lower density outer layers (e.g. Li et al., 2002) may be expected as the inspiral proceeds. As the stream of debris from a partially stripped WD shocks against itself or the inspiraling WD (e.g., Rosswog et al., 2008), some level of electromagnetic emission during such EMRIs is expected. A tidally-triggered detonation of the WD is yet another possibility (Wilson & Mathews, 2004; Dearborn et al., 2005). All these arguments point to the possibility that electromagnetic counterparts may exist for a subset of all the EMRIs detected by LISA (the one corresponding to WD inspirals). The fraction of all EMRIs which involve WDs is, indeed, expected to be substantial (Gair et al., 2004; Hopman & Alexander, 2006a,b).

If electromagnetic counterparts for a subset of all LISA EMRIs can be detected, the rewards will be significant. Just like black hole binary merger events, EMRIs with uniquely identified counterparts and host galaxies could be used to draw a gravitational Hubble diagram, with the significant advantages that, at redshifts as low as  $z \lesssim 0.2-0.3$ , weak-lensing due to line-of-sight inhomogeneities would be small or negligible (Fig. 2) and that the  $D_L$ - $z$  relation at these low redshifts is strongly sensitive to the dominant dark energy content. Dark energy becomes dynamically significant, affecting the expansion rate and geometry of the universe, and modifying  $D_L$ , at  $z \lesssim 1$  (see, e.g. Huterer & Turner, 2001, for a review). Indeed, as emphasized by Hu & Haiman (2003), once cosmic microwave background anisotropies are measured by Planck,  $D_L$  will be known accurately at  $z \sim 1000$ , and a low-redshift ( $z \sim 0$ ) measurement will provide the best complement to constrain dark energy parameters.

WD EMRIs detections with LISA will occur frequently, and they could perhaps be singled out on the basis of the comparatively low mass of the inspiraling compact object. It remains to be determined how the LISA sky localization error evolves with pre-merger (or pre-disruption) time for such events and what is the nature of their electromagnetic counterparts. But altogether, the various advantages that we have outlined point to the need for a detailed assessment of the potential use of WD EMRIs for precision cosmology with LISA.

#### 5. Conclusion

For centuries, astronomers have measured distances exclusively with light. Direct gravitational measurements, gravitational Hubble diagrams and comparisons between the propagation of electromagnetic and gravitational signals offer fundamentally new ways to probe physics on cosmological scales. The novelty involved in joint, time-constrained electromagnetic and gravitational measurements will require that special efforts be made to reach out across the GW and astronomy communities.

This work was supported by NASA grant NNX08AH35G. We thank A. MacFadyen for the permission to use one of his figures.



## References

- Armitage, P. J., & Natarajan, P. 2002, *ApJ*, 567, L9
- Baker, J. G., et al. 2006, *ApJ* 653, L93
- Baker, J. G., et al. 2007, *ApJ*, 668, 1140
- Barack, L. & Cutler, C. 2004, *PRD*, 69, 082005
- Berti, E., Buonanno, A. & Will, C. M. 2005, *PRD* 71, 084025
- Bode, N. & Phinney, S. 2007, APS April Meeting, <http://meetings.aps.org/link/BAPS.2007.APR.S1.10>
- Campanelli, M., Lousto, C. O., Zlochower, Y., & Merritt, D. 2007, *ApJ*, 659, L5
- Chung, D. J., Kolb, E. W., & Riotto, A. 2002, *PRD*, 65, 083516
- Csáki, C., Erlich, J. & Grojean, C. 2001, *Gen. Rel. Grav.* 33, 1921
- Cutler, C. 1998 *PRD*, 57, 7089
- Dalal, N., Holz, D. E., Hughes, S. A. & Jain, B. 2006, *PRD*, 74, 063006
- Dearborn, D. S. P., Wilson, J. R. & Mathews, G. J. 2005, *ApJ*, 630, 309
- Deffayet, C. & Menou, K. 2007, *ApJ*, 668, L143
- Dvali, G., Gabadadze, G. & Porrati, M. 2000, *Phys. Lett. B*, 485, 208.
- Dotti, M., Salvaterra, R., Sesana, A., Colpi, M., & Haardt, F. 2006, *MNRAS*, 372, 869
- Gair, J. R. et al. 2005, *Class. Quant. Grav.* 21, S1595
- Gunnarsson, C., Dahlén, T., Goobar, A., Jönsson, J., & Mörtzell, E. 2006, *ApJ*, 640, 417
- Herrmann, F., Hinder, I., Shoemaker, S., Laguna, P. & Matzner, R. A. 2007, *ApJ* 661, 430
- Holz, D. E., & Hughes, S. A. 2005, *ApJ*, 629, 15
- Hopman, C. & Alexander, T. 2006a, *ApJ* 645, 1152
- Hopman, C. & Alexander, T. 2006b, *ApJL* 645, L133
- Hu, W., & Haiman, Z. 2003, *PRD*, 68, 063004
- Hughes, S. A. 2002, *MNRAS*, 331, 805
- Huterer, D., & Turner, M. S. 2001, *PRD*, 64, 3527
- Ivanov, P. B. & Chernyakova, M. A. 2006, *A&A* 448, 843
- Kocsis, B., Frei, Z., Haiman, Z. & Menou, K. 2006, *ApJ*, 637, 27
- Kocsis, B., Haiman, Z., Menou, K., & Frei, Z. 2007, *PRD*, 76, 2003
- Kocsis, B., Haiman, Z. & Menou, K. 2008, *ApJ* submitted, arXiv:0712.1144
- Kormendy, J. & Richstone, D. 1995, *ARA&A*, 33, 581
- Lang, R. & Hughes, S. A. 2006, *PRD*, 74, 122001
- Lang, R. & Hughes, S. A. 2008, *ApJ* submitted, arXiv:0710.3795
- Li, L.-X., Narayan, R. & Menou, K. 2002, *ApJ*, 576, 753
- Linder, E. 2008, arXiv:0711.0743
- Lippai, Z., Frei, Z. & Haiman, Z. 2008, *ApJL* submitted, arXiv:0801.0739
- Lue, A. 2006, *Phys. Rep.*, 423, 1
- MacFadyen, A. I., & Milosavljevic, M. 2008, *ApJ*, 672, 83
- Magorrian, J., et al. 1998, *AJ* 115, 2285
- Menou, K., Haiman, Z., & Narayanan, V. K. 2001, *ApJ*, 558, 535
- Micic, M., Holley-Bockelmann, K., Sigurdsson, S., & Abel, T. 2007, *MNRAS*, 380, 1533
- Milosavljevic, M., & Phinney, E. S. 2005, *ApJ*, 622, L93
- Rathore, Y., Blandford, R. D. & Broderick, A. E. 2005, *MNRAS*, 357, 834
- Rosswog, S., Ramirez-Ruiz, E. & Hix, W. R. 2008, *ApJ* in press, arXiv:0712.2513
- Schnittman, J. D., & Buonanno, A. 2007, *ApJ*, 662, L63
- Schutz, B. F. 1986, *Nature*, 323, 310
- Sesana, A., Haardt, F., Madau, P. & Volonteri, M. 2004, *ApJ*, 611, 623
- Vecchio, A. 2004, *PRD*, 70, 042001
- Will, C. M. 2006, *Living Rev. Relativity* 9, 3. <http://www.livingreviews.org/lrr-2006-3>
- Wilson, J. R. & Mathews, G. J. 2004, *ApJ*, 610, 368
- Wyithe, J. S., & Loeb A. 2003, *ApJ*, 590, 691